HOMEWORK ASSIGNMENT 10

(DUE DATE: DECEMBER 6, 2018)

Problem 10.1. Let W be the complex vector space of polynomials $P(x, y, z)$ in variables x, y and z. Define the map $T : SL_3(\mathbb{C}) \to GL(W)$ by

$$\left(T \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} P \right) (x, y, z) := P(ax + dy + gz, bx + ey + hz, cx + fy + iz).$$

Let $W_n \subset W$ be the subspace of homogeneous polynomials of degree n. Show that W_n is an irreducible complex representation of $SL_3(\mathbb{C})$.

Problem 10.2. Is the representation $\Lambda^2(ad_{\mathfrak{so}_3(\mathbb{R})})$ of the Lie algebra $\mathfrak{so}_3(\mathbb{R})$ irreducible ?

Problem 10.3. Let T be the standard representation of the Lie algebra $\mathfrak{sl}_2(\mathbb{C})$. Is the representation S^2T irreducible ?

Problem 10.4. We assume known that every finite-dimensional representation of the Lie algebra $\mathfrak{sl}_2(\mathbb{C})$ is completely reducible. Denote by V_n an irreducible $\mathfrak{sl}_2(\mathbb{C})$-module with the highest weight n. Find the decomposition of the representations $V_n \otimes V_m$ into the sum of irreducible ones.

Problem 10.5*. Are the Lie algebras $\mathfrak{sl}_2(\mathbb{C})$ and $\mathfrak{so}_3(\mathbb{C})$ isomorphic ?

Based on: Fulton, Harris, sections 11.1, 11.2, 12.1 Vinberg, sections 10.4-10.9, 11.3, Hall, sections 4.3, 4.6, 4.7 .

Next class: preparation for the final exam.

Date: November 29, 2018.