SUMMARY OF MIDTERM PROBLEMS

Problem MT.1. Does there exist an infinite group with a non-identity element of finite order?

Problem MT.2. Suppose that H is a normal subgroup of a group G, every element of H has finite order, and every element of G/H has finite order. Can there exist an element of infinite order in G?

Problem MT.3. Suppose that the map

$$\phi(x) := x^{-1}$$

of a group G to itself is a homomorphism. Can G be non-abelian?

Problem MT.4. Suppose that the map

$$\phi(x) := x^2$$

of a group G to itself is a homomorphism. Can G be non-abelian?

Problem MT.5. Suppose that H is a normal subgroup of a finite group G, and

$$|G/H| = p,$$

where p is a prime number. Does there always exist an element of order p in G?

Problem MT.6. Show that every element of the group S_n can be represented as a product of transpositions

$$(1,2),(1,3),(1,4),\ldots,(1,n-1),(1,n),$$

where every transposition can enter in the product several times.

Problem MT.7. Does there exist an infinite cyclic group with a non-identity element of finite order?

Problem MT.8. Does there exist a non-abelian group such that its every proper subgroup is cyclic?

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