SAMPLE MIDTERM PROBLEMS

1. Sample Problems

Problem 1.1. *Suppose that a binary operation $(a, b) \mapsto ab$ on G satisfies the following conditions:*

(1) $a(bc) = (ab)c, \quad \forall a, b, c \in G,$

(2) $\exists e \in G$ *such that* $ae = a, \quad \forall a \in G$ *(left identity existence),*

(3) $\forall a \in G \, \exists b \in G$ *such that* $ba = e$ *(right inverse existence).*

Is G always a group?

Problem 1.2. *Describe the center of the group $GL(n, \mathbb{R})$.*

Problem 1.3. *Is every subgroup of $(\mathbb{Q}, +)$ cyclic?*

Problem 1.4. *Let G be an abelian group. Define*

$$H := \{x \in G \mid \text{ord(x) } \textit{is odd}\}. \tag{1.1}$$

Is H always a subgroup of G?

Problem 1.5. *Is it true that every finite group G has a subgroup of order d for any divisor d of $|G|$?*

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