Abstract Algebra I, Spring 2026

Course description. *Algebra holds a unique and foundational place in mathematics, developed extensively from an early stage. At its core, this area of mathematics studies algebraic structures—sets equipped with one or more operations. This course focuses on the essential pillars of abstract algebra: groups, rings, fields, and modules. Modern algebra is a vibrant and dynamic field, with powerful techniques and results that are integral to virtually every area of mathematics. Moreover, its concepts find wide-ranging applications in physics, engineering, and computer science, making the study of algebra both fundamental and highly relevant today.*

Course Program.

1. Properties of division in integers. The GCD and Euclidean algorithm in Z. Bézout's identity. Factoring into primes. Unique factorization in Z .
2. Definition and examples of groups. Abelian and non-abelian groups. Cayley diagrams.
3. Symmetric groups S_n . Cycle notation.
4. Dihedral groups D_n .
5. Definition of a subgroup. The center Z(G) and the derived subgroup G' as examples of subgroups.
6. Order of an element of a group. Cyclic groups. Classification of subgroups of cyclic groups.
7. Cosets, Lagrange's theorem.
8. Group homomorphism. Kernel and image of a homomorphism. Isomorphism of groups.
9. Group action. Orbits, stabilizers. Cayley's Theorem.
10. Inner automorphism. Conjugacy classes.
11. Sign of a permutation. Alternating groups A_n .
12. Normal subgroups. Quotient groups.
13. The first isomorphism theorem. Other isomorphism theorems.
14. Free groups. Groups presented by generators and relations.
15. * The Chinese Remainder Theorem.
16. * The fundamental theorem of finitely generated abelian groups.
17. Definitions of a ring and of an rng (ring without unity). Subrings.

18. Integral domains. Fields. Characteristic of a ring. Examples of finite fields. Fermat's little theorem.
19. Ideals. Quotient ring. Prime and maximal ideals. Principal ideals.
20. Principal ideal domains. Divisibility and factorization in Euclidean domains.
21. Ring of polynomials. Ideals of a polynomial ring. Irreducible polynomials. Eisenstein's irreducibility criterion.
22. Unique factorization in Z[x].
23. Modules over a ring and their properties.
24. * Algebras, division algebras. The algebra of quaternions H . Frobenius theorem classifying finite-dimensional real division algebras.
25. * Extensions of fields.
26. * Symmetric polynomials. Fundamental theorem of algebra

* if time allows

Recommended Textbooks.

D.Dummit, R.Foote, Abstract Algebra.

J.Fraleigh, A First Course in Abstract Algebra.

J.Gallian, Contemporary Abstract Algebra.

C.Pinter, A Book of Abstract Algebra.

Grading Rules. There will be two classes each week: one lecture and one problem-solving class. For each class, we will have a separate Miro board with either lecture notes or a problem set uploaded in advance. Each week you should write solutions to at least four problems total across those boards. You will receive 1 point for each correct solution, up to a maximum of 4 points per week. Only your best 11 weeks' results out of the 12 non-exam weeks (excluding the first week) will count; there is no grading in the first week. In each of the two exam weeks, you should write solutions to at least two problems. Additional bonus points may be awarded for the shortest and most elegant solutions or for solutions to the most challenging exercises.

Both the midterm and final exams are oral, open-book, and open-notes. The midterm consists of 5 problems worth 3 points each, and the final

exam consists of 5 problems worth 4 points each. You also receive 1 bonus point each time you are the first to correct the professor's error during a lecture.

The grading table is as follows:

A+	90%
A	85%
A-	80%
B+	75%
B	70%
B-	65%
C+	55%
C	50%
C-	45%
D	35%

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