PROBLEM SET 3, DUE FEBRUARY 25

3. Linear Maps

Problem 3.1. *Does there exist* $f \in \mathrm{Hom}(\mathbb{R}^3, \mathbb{R})$ *such that*

$$f\left(\begin{pmatrix}-1\\2\\-1\end{pmatrix}\right) = 1, \quad f\left(\begin{pmatrix}3\\-4\\1\end{pmatrix}\right) = 2, \quad f\left(\begin{pmatrix}1\\-1\\0\end{pmatrix}\right) = 1?$$

Problem 3.2. *a) Is it true that the image of a linearly independent subset under a monomorhism has to be linearly independent?*

b) Is it true that the image of a linearly independent subset under an epimorhism has to be linearly independent?

Problem 3.3. *Assume that* $x_1, \ldots, x_n$ *are* n *pairwise distinct real numbers. Consider the map* $A : \mathbb{R}_k[x] \to \mathbb{R}^n$ *defined by*

$$A(p) := \begin{pmatrix} p(x_1) \\ \vdots \\ p(x_n) \end{pmatrix}.$$

Find a basis of the quotient space $\mathbb{R}_k[x]/\ker(A)$*. Find* $\mathrm{rk}(A)$*.*

Problem 3.4. *Suppose that* V *and* U *are finite-dimensional spaces. For which subspaces* W *of the space* V *there exist* $A \in \mathrm{Hom}(V, U)$ *such that* $\ker A = W$*?*

Problem 3.5. *Show that* $A \in \mathrm{Hom}(V, U)$ *is a monomorphism if and only if* $\exists B \in \mathrm{Hom}(U, V)$ *such that* $BA = \mathrm{Id}$*.*

Problem 3.6. *Do there exist vector spaces* V*,* U *and linear maps* $A \in \mathrm{Hom}(V, U)$*,* $B \in \mathrm{Hom}(U, V)$ *such that* $AB = \mathrm{Id}$ *and* $A \notin \mathrm{Iso}(V, U)$*?*

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