PROBLEM SET 9, DUE APRIL 7

9. Jordan Canonical Form Theorem.

Problem 9.1. *Suppose that A is a linear operator in a finite-dimensional vector space V and its characteristic polynomial splits. Is it always true that A is similar to its transpose A^t?*

Problem 9.2. *Suppose that $A, B \in \mathrm{End}(\mathbb{C}^n)$, their characteristic polynomials are equal, and their minimal polynomials are equal. Does A have to be similar to B?*

Problem 9.3. *Find $(J_k(\lambda))^n$.*

Problem 9.4. *For which k and n does there exist $A \in \mathrm{End}(\mathbb{C}^n)$ such that $A^{k+1} = A^k$ and $A^k \neq A^{k-1}$?*

Problem 9.5. *Is it true that $AJ_k(\lambda) = J_k(\lambda)A$ for an $A \in \mathrm{End}(\mathbb{C}^k)$ if and only if $\exists p \in \mathbb{C}[x]$ such that*

$$A = p(J_k(\lambda))?$$

Problem 9.6. *Does there exist a complex square matrix such that every entry is not real and its every eigenvalue is real?*

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