PROBLEM SET 10, DUE APRIL 14

10. Complexification. Jordan Form for the Real Case.

Problem 10.1. *Does there exist* $A \in \mathrm{End}(\mathbb{R}^7)$ *such that* $A^2 + A + \mathrm{Id}$ *is nilpotent?*

Problem 10.2. *Suppose that* $B \in \mathrm{End}(V^{\mathbb{C}})$*. Are the following two conditions equivalent:*

$$(\exists A \in \mathrm{End}(V) \text{ such that } B = A^{\mathbb{C}}) \quad \Longleftrightarrow \quad (B \circ \mathrm{Conj} = \mathrm{Conj} \circ B)?$$

Problem 10.3. *Suppose that* $A \in \mathrm{End}(\mathbb{R}^n)$ *and* $\ker(A^{n-1}) \neq \ker(A^{n-2})$*. What maximum number of distinct eigenvalues can the linear operator* A *have?*

Problem 10.4. *Find* $(J_k(a,b))^n$*.*

Problem 10.5. *Find all* $A \in \mathrm{End}(\mathbb{R}^{2k})$ *such that* $AJ_k(a,b) = J_k(a,b)A$*.*

Problem 10.6. *Find all* $A \in \mathrm{End}(\mathbb{C}^n)$ *such that* $AB = BA$*, where*

$$B = \begin{pmatrix} J_k(\lambda) & 0 & 0 \\ 0 & J_m(\lambda) & 0 \\ 0 & 0 & J_{n-k-m}(\mu) \end{pmatrix}.$$

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