PROBLEM SET 12, DUE APRIL 28

12. Operators in an inner product space.

Problem 12.1. *Suppose that V is a finite-dimensional complex inner product space, $A \in \mathrm{End}(V)$ and*

$$|Ax| = |A^*x|, \quad \forall x \in V.$$

Does A have to be normal?

Problem 12.2. *Suppose that $A \in \mathrm{End}(V)$ is a normal operator in a complex vector space V. Show that $p(A)$ is also normal for every $p \in \mathbb{C}[x]$.*

Problem 12.3. *Suppose that V is a finite-dimensional complex inner product space, A and B are normal linear operators in V, $C \in \mathrm{End}(V)$ and $AC = CB$. Show that $A^*C = CB^*$.*

Problem 12.4. *Do the there exist normal operators $A, B \in \mathrm{End}(V)$ such that $A + B$ is not normal?*

Problem 12.5. *Do the there exist normal operators $A, B \in \mathrm{End}(V)$ such that AB is not normal?*

Problem 12.6. *Let V be a finite-dimensional complex inner product space. Is it true that for every $A \in \mathrm{End}(V)$ there exist self-adjoint operators B and C such that $A = B + iC$?*

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