"Advanced Linear Algebra", Spring 2024.

Course description. It is almost impossible to find an area of pure mathematics that does not use Linear Algebra concepts. The purposes of the course are to cover gaps one may have after a standard course and to add main advanced topics: tensor products, complexification, and Hilbert spaces. As a result, we hope to prepare our students for every subject (including graduate courses) that requires Linear Algebra knowledge. No prerequisites.

Program of the Course.

1. Groups. Fields. Vector spaces. Bases, Dimension. Subspaces, quotient spaces. ([FIS], 1.1 – 1.6, [A], ch.1, 2.)
2. Linear maps. Matrices. Multiplication of matrices. Change of basis. ([FIS], 2.1 – 2.5, [A], ch.3.)
3. Dual space. Tensor product of vector spaces, universality property. Symmetric tensors, skew-symmetric tensors. Tensor product of linear operators, its matrix. ([R], ch.14, [FIS], 2.6.)
4. Determinant. Systems of linear equations. Inverse operator. ([FIS], 3.1 – 3.4, 4.1 – 4.4, [A], ch.10.)
5. Characteristic polynomial of an operator. Cayley-Hamilton theorem. ([FIS] 5.1,5.2,5.4, [A], ch.5.)
6. Eigenvectors and eigenvalues. Jordan Canonical Form (complex case). ([FIS], 7.1 – 7.3, [A], ch.8.)
7. Complexification. Jordan form (real case). ([A], ch.9.)
8. Real inner product spaces, complex inner product spaces. ([FIS], 6.1, 6.2, [A], ch.6.)
9. Spectral theory for normal operators, unitary operators, Hermitian operators, skew-Hermitian operators, orthogonal operators, symmetric operators, skew-symmetric operators. ([FIS], 6.3 -6.6, [A], ch.7.)
10. Bilinear forms. The matrix of a bilinear form. Generalization of orthogonality notion.([R], ch.11, [FIS], 6.7)
11. Positive operators. Sylvester's Criterion. Polar decomposition. ([A], ch.7.)
12. Hilbert spaces, Hilbert bases. The Riesz Representation Theorem. ([R], ch. 13)

Recommended Textbooks.

- [FIS] S. Freidberg, A. Insel, L. Spence, Linear Algebra.
- [A] S. Axler, Linear Algebra Done Right.
- [R] S. Roman, Advanced Linear Algebra.

Grading Rules. Each week we will have two classes: the lecture and the discussion of problem sets. The lecture notes will be available in advance before each lecture. They contain plenty of exercises. You need to solve at least three of them for one point each. You are encouraged to solve more but you won't get credit for them. After each lecture, I

will upload a list of problems. Again you need to solve at least three for one point each. In both cases, the due time is an hour before the beginning of the next class. You may write your solutions on the corresponding class whiteboard or email me directly. The midterm and the final are oral exams, open books, and open notes. The midterm contains 5 problems for 4 points each, and the final – has 5 problems for 6 points each. You may get bonus points for creative solutions or for tackling challenging questions. Also, you will get 1 bonus point every time you are the first to correct the professor's mistake/mistype during the lecture. The grading table has the following form:

A+	90%
A	85%
A-	80%
B+	70%
B	65%
B-	60%
C+	50%
C	45%
C-	40%
D	35%

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