PROBLEM SET 2

2. Subspaces, Direct Sums

Problem 2.1. *Is the set*

$$\left\{ \begin{pmatrix} a \\ a+1 \\ b \end{pmatrix} \mid a, b \in \mathbb{R} \right\} \tag{2.1}$$

a subspace of the vector space $\mathbb{R}^3$*?*

Problem 2.2. *Is the set*

$$\left\{ \begin{pmatrix} a+3b \\ a \\ b-a \end{pmatrix} \mid a, b \in \mathbb{R} \right\} \tag{2.2}$$

a subspace of the vector space $\mathbb{R}^3$*?*

Problem 2.3. *Is the set*

$$\left\{ \begin{pmatrix} 0 \\ ab \\ 3b \end{pmatrix} \mid a, b \in \mathbb{R} \right\} \tag{2.3}$$

a subspace of the vector space $\mathbb{R}^3$*?*

Problem 2.4. *Is the set*

$$\left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} \mid a, b, c \in \mathbb{R},\ 2a + c = 1 \right\} \tag{2.4}$$

a subspace of the vector space $\mathbb{R}^3$*?*

Problem 2.5. *Is the set*

$$\left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} \mid a, b, c \in \mathbb{R},\ a - 2b = 0 \right\} \tag{2.5}$$

a subspace of the vector space $\mathbb{R}^3$*?*

Problem 2.6. *Is the set*

$$\left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} \mid a, b, c \in \mathbb{R},\ a + 2b + c = 0 \right\} \bigcup \left\{ \begin{pmatrix} d \\ 0 \\ -d \end{pmatrix} \mid d \in \mathbb{R} \right\} \tag{2.6}$$

a subspace of the vector space $\mathbb{R}^3$*?*

Problem 2.7. *Suppose that* U_1 *and* U_2 *are subspaces of a vector space* V*. Show that the set* $U_1 \cup U_2$ *is also a subspace of* V *iff* $U_1 \subset U_2$ *or* $U_2 \subset U_1$*.*

Problem 2.8. *Define*

$$U_1 := \left\{ \begin{pmatrix} a+b \\ a-b \\ 0 \end{pmatrix} \mid a, b \in \mathbb{R} \right\}, \quad U_2 := \left\{ \begin{pmatrix} c \\ d \\ e \end{pmatrix} \mid c, d, e \in \mathbb{R},\ c + d + e = 0 \right\} \tag{2.7}$$

Is it true that

$$\mathbb{R}^3 = U_1 \oplus U_2? \tag{2.8}$$

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Problem 2.9. *Define*

$$U_1 := \left\{ \begin{pmatrix} a+b \\ 2(a+b) \\ 0 \end{pmatrix} \mid a, b \in \mathbb{R} \right\}, \quad U_2 := \left\{ \begin{pmatrix} c \\ d \\ e \end{pmatrix} \mid c, d, e \in \mathbb{R},\ c+d+e=0 \right\} \tag{2.9}$$

Is it true that

$$\mathbb{R}^3 = U_1 \oplus U_2? \tag{2.10}$$

Problem 2.10. *Recall that $\mathbb{R}_2[x]$ stands for the vector space of real polynomials in x of degree not greater than 2. Define*

$$U_1 := \{p \in \mathbb{R}_2[x] \mid p(0) = p(-1) = 0\}, \quad U_2 := \{p \in \mathbb{R}_2[x] \mid p(1) = 0\}. \tag{2.11}$$

Is it true that

$$\mathbb{R}^3 = U_1 \oplus U_2? \tag{2.12}$$