PROBLEM SET 9

9. The Jordan Canonical Form Theorem

Problem 9.1. *Find* $(J_k(\lambda))^n$.

Problem 9.2. *Suppose that A is a linear operator on a finite-dimensional vector space V and its characteristic polynomial splits. Is it always true that A is similar to its transpose A^t?*

Problem 9.3. *Suppose that $A, B \in \mathrm{End}(\mathbb{C}^n)$ have equal characteristic polynomials and equal minimal polynomials. Does A have to be similar to B?*

Problem 9.4. *For which k and n does there exist an $A \in \mathrm{End}(\mathbb{C}^n)$ such that $A^{k+1} = A^k$ and $A^k \neq A^{k-1}$?*

Problem 9.5. *Suppose that $A \in \mathrm{End}(\mathbb{C}^n)$ and $\mathrm{Im}(A^{k+1}) \neq \mathrm{Im}(A^k)$. For which pairs (k, n) do those conditions imply that A is nilpotent?*

Problem 9.6. *Is it true that for an $A \in \mathrm{End}(\mathbb{C}^k)$, $AJ_k(\lambda) = J_k(\lambda)A$ if and only if $\exists p \in \mathbb{C}[x]$ such that*

$$A = p(J_k(\lambda))?$$

Problem 9.7. *Find all $A \in \mathrm{End}(\mathbb{C}^n)$ such that $AB = BA$, where*

$$B = \begin{pmatrix} J_k(\lambda) & 0 & 0 \\ 0 & J_m(\lambda) & 0 \\ 0 & 0 & J_{n-k-m}(\mu) \end{pmatrix}.$$

Problem 9.8. *Does there exist a complex square matrix such that no entry is real and all its eigenvalues are real?*

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