Advanced Linear Algebra, Spring 2026

Course description. *It is almost impossible to find an area of pure mathematics that does not use linear algebra concepts. The purposes of the course are to cover gaps one may have after a standard course and to add main advanced topics: tensor products, complexification, and Hilbert spaces. As a result, we hope to prepare our students for every subject (including graduate courses) that requires linear algebra knowledge.*
The course is designed to foster a collaborative theorem-proving approach. Several days before each class, you will receive lecture notes containing definitions, examples, and propositions to be proved. During class, we will engage in discussions around your approaches, insights, and questions to deepen your understanding and facilitate active participation. No prerequisites.

Course Program

1. Groups, fields, vector spaces. Subspaces, direct sums, quotient spaces.

2. Linear span and linear independence. Bases, dimension.

3. Types of linear maps. Image and kernel of a linear map. The First Isomorphism Theorem. Invertibility and isomorphisms.

4. Matrices, multiplication of matrices. Change of basis. Systems of linear equations. Dual space.

5. Tensor product of vector spaces, universality property. Symmetric group. Symmetric tensors, skew-symmetric tensors.

6. Tensor product of linear maps and its matrix. Invariant definition of the determinant.

7. Eigenvectors and eigenvalues. Characteristic polynomial of an operator. Cayley-Hamilton theorem.

8. Primary Decomposition Theorem, Jordan Canonical Form (complex case).

9. Complexification, Jordan Canonical Form (real case).

10. Real inner product spaces, complex inner product spaces. Orthogonal projection, Fourier expansion, Bessel's inequality, Parseval's identity. The Riesz Representation Theorem.

11. Spectral theory for normal operators in an inner product space. Unitary operators, Hermitian operators, skew-Hermitian operators, orthogonal operators.

12. * Positive operators, polar decomposition.

13. * Hilbert spaces, Hilbert bases.

(* - if time allows)

Recommended Textbooks.

S. Friedberg, A. Insel, L. Spence, Linear Algebra, 5th edition, 2018.

S. Axler, Linear Algebra Done Right, 4th edition, 2024.

S. Roman, Advanced Linear Algebra, 3rd edition, 2008.

Grading Rules. There will be two classes each week: one lecture and one problem-solving class. For each class, we will have a separate Miro board with either lecture notes or a problem set uploaded in advance. Each week you should write solutions to at least four problems total across those boards. You will receive 1 point for each correct solution, up to a maximum of 4 points per week. Only your best 11 weeks' results out of the 12 non-exam weeks (excluding the first week) will count; there is no grading in the first week. Additional bonus points may be awarded for the shortest and most elegant solutions or for solutions to the most challenging exercises.

Both the midterm and final exams are oral, open-book, and open-notes. The midterm consists of 5 problems worth 3 points each, and the final exam consists of 5 problems worth 4 points each. You also receive 1 bonus point each time you are the first to correct the professor's error during a lecture.

The grading table is as follows:

A+	90%
A	85%
A-	80%
B+	75%
B	70%
B-	65%
C+	55%
C	50%
C-	45%
D	35%

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