FINAL EXAM SAMPLE PROBLEMS

Problem F.1. Find the Taylor polynomial of the fourth degree with center at $x_0 = 0$ of the function $f(x) := (x - x^2)\ln(1 - x)$.

Problem F.2. Find the limit

$$\lim_{x\to 0^+} \frac{x((\ln(x^2 + x))^2 - 1)}{(x - 1)^3}.$$

Problem F.3. Does there exist a function $f \in C(a, b)$ that is not uniformly continuous on (a, b)?

Problem F.4. Let

$$f := \begin{cases} 1, & \text{if } x = \frac{1}{n}, n \in \mathbb{Z}_+; \\ 0, & \text{otherwise.} \end{cases}$$

Is f Riemann integrable on $[0, 1]$?

Problem F.5. For which $a \in \mathbb{R}$ does the limit

$$\lim_{y\to 0^+} \int_y^1 x^a dx$$

exist?

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