MIDTERM EXAM SAMPLE PROBLEMS

Problem MT.1. Show that

$$\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n=\frac{1}{e}.$$

Problem MT.2. Does there exist a continuous function $f:\mathbb{R}\to\mathbb{R}$ such that f is not differentiable at the points $\frac{1}{n}$ for every positive integer n?

Problem MT.3. Suppose that $f:\mathbb{R}\to\mathbb{R}$ is a differentiable function, $f(0)=1$, $f(x)>0,\forall x\in\mathbb{R}$ and $f'(x)=f(x),\forall x\in\mathbb{R}$. Show that

$$f(x)=e^x.$$

Problem MT.4. Suppose that $f:\mathbb{R}\to\mathbb{R}$ is a differentiable function, $c\in\mathbb{R}_+$, $f(0)=0$ and $|f'(x)|\le c$, $\forall x\in\mathbb{R}$. Show that

$$|f(x)|\le c|x|\,\forall x\in\mathbb{R}.$$

Problem MT.5. Suppose that $f:\mathbb{R}\to\mathbb{R}$ is a function, $f\in D^1(\mathbb{R}\setminus\{0\})$ and the limit

$$\lim_{x\to 0}f'(x)=L$$

exists. Must f be differentiable at 0?

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