"Analysis I", Spring 2025.

Course description. This course is a natural continuation of our Foundations of Analysis course. A rigorous understanding of real numbers and sequence limits enables us to study key concepts in Real Analysis: convergence of series, subsets of the real line, continuity of functions.

We have structured the course to provide you with opportunities to collaboratively prove all theorems with your peers. Several days before each class, you will receive lecture notes containing definitions, examples, and propositions that we need to prove. In class, we will discuss your approaches, comments, and any questions you may have.

Program of the Course.

1. Limit points of sequences. Subsequences.
2. Limit superior and limit inferior.
3. Nested Intervals Property.
4. Completeness of the set of real numbers.
5. Real exponentiation.
6. Sequences, defined by recurrence relations.
7. Infinite series. Absolute and conditional convergence.
8. Cauchy Criterion for series. The harmonic series.
9. The root and ratio tests.
10. Subsets of the real line. Open and closed subsets.
11. Closure. Adherent and limit points of a set.
12. The Heine-Borel theorem for the line.
13. Limiting values of functions. Left and right limits.
14. Continuous functions. Singularities.
15. Dirichlet's and Thomae's functions.
16. The maximum principle.
17. The intermediate value theorem.
18. Monotonic functions.
19. Limits at infinity.
20. Compact sets. The Cantor set. The Heine-Borel covering theorem.

Recommended Textbooks.

T.Tao, Analysis I.

V.Zorich, Mathematical Analysis I.

W. Rudin, Principles of Mathematical Analysis.

S.Abbott, Understanding Analysis.

Grading Rules. You should write a solution to at least two problems before each seminar and two exercises before each lecture. You will get 1 point for every correct solution but no more than two points for each lecture or seminar. Additional bonus points may be awarded for the shortest and most elegant solutions or solutions for the most complicated exercises. Both the midterm and the final exam are oral exams, open books, and open notes. The midterm contains 5 problems for 3 points each, and the final exam – 5 problems for 4 points each. Also, you get one bonus point every time you are the first to correct the professor's error during the lecture. The grading table has the following form:

A+	90%
A	85%
A-	80%
B+	75%
B	70%
B-	65%
C+	55%
C	50%
C-	45%
D	35%

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