**"Basic Representation Theory", Fall 2022.**

***Course description.*** Welcome to the Basic Representation Theory course! Representation Theory studies how a given group may act in vector spaces. It is a fundamental tool to study groups using linear algebra. But even more importantly the structure of representations allows to construct basic mathematical models in Mathematical Physics and other applications. The course introduces basic concepts and results of the classical theory of complex representations of finite groups and simple examples of representations of Lie groups and Lie algebras.

***Program of the Course.***

0. Alongside the discussion of representations we are going to recall some (multi)linear algebra concepts and theorems: Jordan canonical form theorem, bilinear forms, inner product spaces, spectral theorem for unitary operators, tensor products of vector spaces, symmetric and exterior algebras.
1. Linear representations of groups. Definitions and examples. Irreducible representations. Schur's Lemma. Complete reducibility.
2. Characters of representations. Number of irreducible characters. Character tables and orthogonality relations. Group algebra.
3. First examples of character tables: abelian groups, dihedral group D_n , groups S_3 , S_4 , A_4.
4. Decomposition of tensor products and symmetric powers of irreducible representations.
5. * Representations of symmetric groups, Young diagrams.
6. Examples of Lie groups and Lie algebras. Covering of SO(3,R) by SU(2).
7. Representations of Lie algebra sl(2,C). Clebsch-Gordan decomposition.
8. Compact groups and their representations. Peter-Weyl theorem.
9. Connection between representations of Lie groups and Lie algebras.
10.* Complex simple Lie algebras and their irreducible representations.

* - if time allows

***Recommended Textbooks.***

1. W. Fulton, J. Harris, Representation Theory. A First Course.
2. E.B. Vinberg, Linear Representations of Groups.
3. G.James, M.Liebeck, Representations and Characters of Groups. (for items 1-5 of the program)
4. B.C. Hall, Lie Groups, Lie Algebras, and Representations. (for items 6-10 of the program)

***Grading Rules.*** During the semester you will receive about 11 multiple choice quizzes, each consisting of 4-7 questions. For a correct answer you get one point. Also you should write a solution of at least one problem before each seminar and you will get 1 point for your attempt. Additional bonus points may be awarded for shortest and most elegant solutions or solutions of the most complicated exercises. Both the midterm and the final exam are oral exams, open books, open notes. The midterm contains 5 problems for 3 points each, and the final – 5 problems for 4 points each. Also you get 1 bonus point every time you are the first to correct the professor's mistake/mistype during the lecture. The grading table has the following form:

A+ 90%

A 85%

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| A- | 80% |
| B+ | 70% |
| B | 65% |
| B- | 60% |
| C+ | 55% |
| C | 50% |
| C- | 45% |
| D | 35% |

***Contact information.*** vivanov (at) mathinmoscow.org