PROBLEM SET 4, DUE OCTOBER 12

4. Orthogonality relations.

Problem 4.1. *Construct the complex character table of the group A_4.*

Problem 4.2. *Construct the complex character table of the group S_4.*

Problem 4.3. *Construct the complex character table of the group Q_8.*

Problem 4.4. *Describe the conjugacy classes of the group D_n.*

Problem 4.5. *Construct the complex character table of the group D_n when n is odd.*

Problem 4.6. *Construct the complex character table of the group D_n when n is even.*

Problem 4.7. *Find the multiplicities of the decomposition of every tensor product of two irreducible complex representations (they may be equal) of the group A_4 into the sum of irreducible ones.*

Problem 4.8. *Find the multiplicities of the decomposition of every tensor product of two irreducible complex representations (they may be equal) of the group S_4 into the sum of irreducible ones.*

Problem 4.9. *For every irreducible complex representation T of the group S_4 find the multiplicities of the decomposition of the representations S^2T and $\Lambda^2 T$ into the sum of irreducible ones.*

Problem 4.10. *Consider a set F of complex functions on the set M of the faces of the cube. It is clear that F is a vector space with respect to the addition and the multiplication by complex numbers defined by the following formulas:*

$$(\lambda \cdot f)(x) := \lambda(f(x)), \qquad (f+g)(x) := f(x) + g(x), \qquad \forall f, g \in F, \forall x \in M, \forall \lambda \in \mathbb{C}.$$

Recall that $S_4 \cong SO(C)$, which is the group of orientation-preserving isometries of the cube. For arbitrary elements $g \in S_4$, $x \in M$ denote by gx the image of the face x after applying the rotation from $SO(C)$ corresponding to g. Denote by L a representation of the group S_4 in the vector space F given by the following formula

$$(L(g)f)(x) := f(g^{-1}x), \qquad \forall f \in F, \forall x \in M, \forall g \in S_4.$$

Find the multiplicities of the decomposition of the representation L into the sum of irreducible ones.

Problem 4.11. *Describe minimal invariant subspaces of the representation L from the previous problem.*

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