

Basic Algebra

Semester: Fall 2020

Instructor: Alexander Pavlov

Course outline:

1. Basics of groups.
Definition of a group and subgroup. Cayley tables. Group homomorphisms. Kernel and image subgroups. Product of groups. Subgroups of the additive group of integers.
2. Examples of groups.
Cyclic groups. Dihedral groups. Quaternion group Q_8 . Symmetric groups. Free groups.
3. Cosets and quotient groups.
Equivalence relations and partitions. Definition of left and right cosets. Normal subgroups and normalizers. Quotient groups. Generators and relations. Lagrange's theorem and its corollaries. Modular arithmetic. Fermat's little theorem. Euler's function and theorem.
4. Isomorphism theorems.
Four isomorphism theorems for groups.
5. Group actions.
Definitions. Orbits, stabilizers and kernels of the action. Cycle decomposition of permutations. Cayley theorem. Cauchy theorem. Action by conjugation and the class equation. Burnside's formula: application to counting problems with symmetries.
6. Basics of rings.
Definition of a ring, subring and ring homomorphism. Integral domains. Fields.
7. Quotient rings.
Ideals and quotient rings. Isomorphism theorems. Prime ideals and maximal ideals.
8. Euclidian domains and principal ideal domains.
Division algorithm in Euclidian domains. Euclidian domains are principal ideal domains.
9. Fields and vector spaces.
Definition of a vector space. Subspaces. Linear dependence. Basis and dimension. Field extensions. Finite fields.
10. Algebras over fields.
Matrix algebra. Matrix groups. Quaternions. Exterior algebra and determinant.

Recommended textbooks

Gallian J. A. Contemporary abstract algebra, 9th ed.

Artin M. Algebra, 2nd ed.

Dummit D. S., Foote R. M. Abstract Algebra 3rd ed.

Grading policy: home assignments 20% ; midterm 30%; final 50%