

## Commutative algebra. Problems. Set 2.

1. Prove Theorem 1.4 from the lecture notes.
2. Prove that  $(I_1 \cap I_2)^e = I_1^e \cap I_2^e$ .
3. Suppose  $A_1$  and  $A_2$  are two rings. Describe prime ideals of the ring  $A = A_1 \times A_2$ .
4. Suppose  $A_0$  is a ring,  $A = A_0[T]$ ,  $P = a_0 + a_1T + \cdots + a_nT^n \in A$  a polynomial. Prove that:
  - 1)  $P$  is nilpotent  $\Leftrightarrow$  all the  $a_i$  are nilpotent.
  - 2)  $P$  is invertible  $\Leftrightarrow$   $a_0$  is invertible in  $A_0$  and all other  $a_i$ 's are nilpotent.
5. Let  $A_0$  be a ring,  $A = A_0[T]$ . Prove that  $\mathfrak{J}(A) = \mathfrak{R}(A)$ .