ADDENDUM TO LECTURES 4 AND 8

Please consider every proposition in this text to be an exercise.

1. Subsets of natural numbers.

Proposition 1.1. *Every non-empty subset of the set $\mathbb{N}$ has a minimal element.*

Proposition 1.2. *Every non-empty upper-bounded subset of the set $\mathbb{N}$ has a maximal element.*

Proposition 1.3. *Every lower-bounded non-empty subset of $\mathbb{Z}$ has a minimal element.*

2. Properties of the absolute value function

Proposition 2.1. *Suppose that the elements x and y are arbitrary rational numbers. Then we get the following equalities and inequalities:*

$$|-x| = |x|, \tag{2.1}$$
$$|x+y| \le |x| + |y|, \tag{2.2}$$
$$|x-y| \ge |x| - |y|, \tag{2.3}$$
$$|x+y| \ge |x| - |y|, \tag{2.4}$$
$$|xy| = |x| \cdot |y|, \tag{2.5}$$
$$\left|\frac{1}{x}\right| = \frac{1}{|x|}. \tag{2.6}$$

Lemma 2.2. *Suppose that $x \ge a > 0$ and $|x-y| < b$. Then*

$$y > a - b. \tag{2.7}$$

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