"Foundations of Analysis", Fall 2024.

Course description. Welcome to the Foundations of Analysis course! Real Analysis has developed from an ingenious definition of the limit. This subject is a cornerstone of modern mathematics. The course offers a comprehensive guide to this area. We begin with the fundamentals – the definition of real numbers and delve into various subtle convergence problems. No prerequisites.

Program of the Course.

1. Sets, maps, operations with them. Cardinality of sets.
2. The Peano axioms. The unique factorization theorem for integers. The rationals.
3. Countable and uncountable sets. Cantor's Theorem.
4. Cauchy sequences. The construction of the real numbers. The least upper bound property, the Archimedean property, and the nested interval property.
5. Convergence and limit laws. The squeeze test.
6. The Bolzano–Weierstrass theorem. The Heine-Borel covering theorem.
7. Real exponentiation. The number e.
8. Infinite series. Rearrangement of series. The root and ratio tests.
9. Fubini's theorem.*
10. Open and closed sets. Compact Sets. The Cantor Set. Baire's Theorem.
11. Limiting values of functions. Continuous functions. Singularity. Dirichlet's and Thomae's functions. The maximum principle. The intermediate value theorem.
12. Derivative of a function. Newton's approximation. The Leibniz rule. Chain rule. Inverse functions.
13. Local maxima and local minima. Rolle's theorem. Mean value theorem. Taylor series.
14. L'Hospital's rule. Darboux's theorem.*
15. The Riemann integral. Riemann integrability of continuous functions.
16. Lebesgue's Criterion for Riemann Integrability.*
17. The Riemann-Stieltjes integral. The two fundamental theorems of calculus.
18. Sequences of functions. Pointwise and uniform convergence. The Weierstrass M-test. Integration and derivatives of a uniformly convergent sequence of functions.
19. Power series, Abel's test theorem.
20. The Arzela–Ascoli theorem.*
21. The Stone-Weierstrass theorem.*

(* – if time allows)

Recommended Textbooks.

S.Abbott, Understanding Analysis.
T.Tao, Analysis I.
V.Zorich, Mathematical Analysis I.
W. Rudin, Principles of Mathematical Analysis.
T. Apostol, Mathematical Analysis.
R. Strichartz, The Way of Analysis.

Grading Rules. You should write a solution to at least two problems before each seminar and two exercises before each lecture. You will get 1 point for every correct solution but no more than two points for each lecture or seminar. Additional bonus points may be awarded for the shortest and most elegant solutions or solutions for the most complicated exercises. Both the midterm and the final exam are oral exams, open books, and open notes. The midterm contains 5 problems for 3 points each, and the final – 5 problems for 4 points each. Also, you get one bonus point every time you are the first to correct the professor's mistake/mistype during the lecture. The grading table has the following form:

A+	90%
A	85%
A-	80%
B+	75%
B	70%
B-	65%
C+	55%
C	50%
C-	45%
D	35%

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