"Foundations of Analysis", Fall 2026.

Course description. Welcome to the Foundations of Analysis course! Our primary goal is to establish the rigorous foundations upon which all of analysis is built. We begin at the very beginning: the Peano axioms, which give us the natural numbers. From there, we construct the integers and the rational numbers through set-theoretic constructions, culminating in the construction of the real numbers. With this solid foundation in place, we will be ready to explore the definition of the limit of a sequence. This concept, which revolutionized 19th-century mathematics, serves as the gateway to modern abstract analysis.

Program of the Course.

1. Sets, functions, set operations (union, intersection, etc.), composition of functions. Image and preimage. Injectivity, surjectivity.
2. The Peano axioms. Addition and multiplication of natural numbers. The ordering of the natural numbers.
3. Well-ordering principle.
4. Construction of the integers as equivalence classes of pairs of natural numbers; addition, multiplication, ordering.
5. Fundamental Theorem of Arithmetic for natural numbers.
6. Construction of the rational numbers; addition, multiplication, ordering.
7. Cauchy sequences of rational numbers.
8. Definition of real numbers. Addition and multiplication.
9. The ordering of the real numbers.
10. The least upper bound property. The Archimedean property.

11. Definition of the limit of a sequence. Cauchy sequences. Completeness of the set of real numbers.
12. Algebraic properties of limits of sequences.
13. Real exponentiation.
14. Subsequences. Limit points of sequences.
15. Nested Intervals Property. The Bolzano–Weierstrass theorem.
16. Limit superior and limit inferior.

Recommended Textbooks.

S.Abbott, Understanding Analysis.
T.Tao, Analysis I.
V.Zorich, Mathematical Analysis I.
W. Rudin, Principles of Mathematical Analysis.

Grading Rules. Each week you should write solutions to at least three problems total across the Miro boards of our course, the preparatory boards do not count. You will receive 1 point for each correct solution, up to a maximum of 3 points per week. There is no grading in the first week. In each of the two exam weeks, you should write solutions to at least two problems. Additional bonus points may be awarded for a very elegant solutions or for solutions to the most challenging exercises.

Both the midterm and final exams are oral, open-book, and open-notes. The midterm consists of 5 problems worth 3 points each, and the final exam consists of 5 problems worth 4 points each. You also receive 1 bonus point each time you are the first to correct the professor's error during a lecture.

A+	90%
A	85%
A-	80%
B+	75%
B	70%
B-	65%
C+	55%
C	50%
C-	45%
D	35%

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