**MATH IN MOSCOW. COMMUTATIVE ALGEBRA.**
**HOMEWORK 3**

(1) Let $f\colon A \to B$ be a ring homomorphism. Let $M$ be a flat $A$-module. Show that $M_B = B \otimes_A M$ is a flat $B$-module.

(2) Let $A$ be a local ring, and let $M$ and $N$ be finitely generated $A$-modules. Assume that $M \otimes N = 0$. Prove that either $M = 0$ or $N = 0$.

(3) Assume that $M$ and $N$ are flat $A$-modules. Prove that $M \otimes N$ is also flat.

(4) Let $A$ be a ring. Assume that $A^n \simeq A^m$ for some $n, m \geq 1$. Prove that $n = m$. Prove that if a homomorphism $A^n \to A^m$ is surjective then $n \geq m$. Is it true that if a homomorphism $A^n \to A^m$ is injective then $n \leq m$?

(5) Let $M_i, i \in I$ be a family of $A$-modules. Prove that their direct sum is flat if and only if each $M_i$ is flat.

(6) Make precise and prove the following claim: tensor product commutes with direct limits.