MATH IN MOSCOW. COMMUTATIVE ALGEBRA.
HOMEWORK 4

(1) Let $A = \mathbb{K}[x, y]$ be a polynomial ring over a field. Is $B = A/(x)$ a flat A-module?

(2) Assume that an A-module M is of finite presentation, that is, there exists an exact sequence

$$A^m \to A^n \to M \to 0.$$

Let $S \subset A$ be a multiplicative subset. Prove that

$$S^{-1}(\mathrm{Hom}_A(M, N)) \simeq \mathrm{Hom}_{S^{-1}A}(S^{-1}M, S^{-1}N).$$

(3) Assume that A is a domain, so that any localization in a prime ideal $A_{\mathfrak{p}}$ can be considered as a subring of the field of fractions $\mathrm{Frac}(A)$. Prove that

$$A = \bigcap_{\mathfrak{p}} A_{\mathfrak{p}}.$$

(4) Describe prime ideals in $S^{-1}A$ in terms of the prime ideals in A.

(5) Prove that the operation S^{-1} for the ideals of a ring A commutes with direct sums, products, intersections and taking radicals.

(6) Prove that $S^{-1}(\mathrm{Nil}(A)) = \mathrm{Nil}(S^{-1}A)$.