MATH IN MOSCOW. COMMUTATIVE ALGEBRA.
HOMEWORK 6

(1) Assume that for an ideal $I \subset A$ we have that $R(I) = I$. Prove that in the primary decomposition of I there are no embedded components.

(2) Consider the ideals $M = (2, t)$ and $I = (4, t)$ in the ring $A = \mathbb{Z}[t]$. Prove that M is maximal, I is M-primary but is not a power of M.

(3) Consider the ideals $I_1 = (x, y)$, $I_2 = (x, z)$, $I_3 = (x, y, z)$ in the ring $\mathbb{K}[x, y, z]$. Put $I = I_1 I_2$. Write down a primary decomposition of I in terms of I_i. Which components of this decomposition are minimal? Which are embedded?

(4) Prove that in a polynomial ring $\mathbb{K}[x_1, \ldots, x_n]$ ideals of the form $(x_1, \ldots, x_k)$, $1 \leq k \leq n$ are prime, and their powers are primary.

(5) Describe a correspondence between primary ideals in A and primary ideals in $S^{-1}A$ where $S \subset A$ is a multiplicative subset.

(6) Assume that an ideal $I \subset A$ admits a primary decomposition. Let $S \subset A$ be a multiplicative subset. Describe a primary decomposition of $S^{-1}I$ in $S^{-1}A$ in terms of a primary decomposition of I in A.