MATH IN MOSCOW. COMMUTATIVE ALGEBRA.
HOMEWORK 7

(1) Let $P \subset A$ be a prime ideal. Compute $\mathrm{Ass}(A/P)$. Prove that if A is Noetherean and M is an A-module then $\mathrm{Ass}(M)$ is finite.

(2) Let $A = \mathbb{K}[x, y]$, and let $M = A/(xy, x^2)$. Compute $\mathrm{Ass}(M)$.

(3) Let M be an A-module. Define the *support* of an A-module M as the set

$$\mathrm{Supp}(M) = \{P \text{ is a prime ideal in } A \mid M_P \neq 0\}.$$

Prove that $\mathrm{Ass}(M) \subset \mathrm{Supp}(M)$.

(4) Consider a $\mathbb{Z}$-module $M = \bigoplus_{p>2} \mathbb{Z}/p$ where p is a prime number. Compute $\mathrm{Supp}(M)$.

(5) Let $A = \mathbb{K}[x_1, x_2, \ldots]/I$ where $I = (x_i^{i+1} | i \geq 1)$. Prove that $\mathrm{Ass}(A)$ is empty.

(6) Let $A = \mathbb{K}[x, y]$. Fix any maximal ideal $\mathfrak{m} \subset A$ such that $A/\mathfrak{m} \simeq \mathbb{K}$. Let $M = \mathrm{Hom}_{\mathbb{K}}(R, R/M)$. Prove that $\mathfrak{m}$ is an associated prime of M.