MATH IN MOSCOW. COMMUTATIVE ALGEBRA.
HOMEWORK 9

(1) Prove that a short exact sequence of complexes induce a long exact sequence of cohomology (as stated in the class).

(2) A morphism of complexes $f^\bullet\colon M^\bullet \to N^\bullet$ is called a *quasi-isomorphism* if it induces an isomorphism in cohomology. Prove that any complex is quasi-isomorphic to a complex whose maps are 0.

(3) Find a maximal length of a regular sequence of elements in $\mathbb{K}[x]/(x^2)$, $\mathbb{K}[x,y]/(x^2,xy)$, $\mathbb{K}[x^2,x^3]$.

(4) Assume that $N = N' \oplus N''$. Prove that $\Lambda^\bullet N = \Lambda^\bullet N' \otimes \Lambda^\bullet N''$.

(5) Let R be a ring. Consider an exact sequence of complexes

$$0 \to R[-1] \to F^\bullet \to R \to 0$$

where $F^\bullet$ is the Koszul complex of the form

$$0 \to R \to^{\cdot y} R \to 0.$$

Prove that the connecting homomorphism in the associated long exact sequence of cohomology is a multiplication by y map.