MATH IN MOSCOW. ALGEBRAIC GEOMETRY. HOMEWORK 2

(1) Let X be affine variety given by the equation $y^2 - x^3 = 0$ in $\mathbb{A}^2$. Consider a regular map $f\colon \mathbb{A}^1 \to X$ given by $f(t) = (t^2, t^3)$. Prove that f is not an isomorphism. Is it a bijection?

(2) An isomorphism $f\colon X \to X$ is called an *automorphism* of X. Classify automorphisms of $X = \mathbb{A}^1 - \{0\}$.

(3) Classify prime and maximal ideals in $\mathbb{Z}/n\mathbb{Z}$ and $\mathbb{C}[x]$.

(4) Construct an example of a regular map between affine algebraic sets whose image is neither open nor closed.

(5) Let X be given by the equation $xy - 1 = 0$ in $\mathbb{A}^2$. Prove that X is an affine variety not isomorphic to $\mathbb{A}^1$.