MATH IN MOSCOW. ALGEBRAIC GEOMETRY. HOMEWORK 3

(1) Let I_1, I_2 be ideals in a ring R. Prove that $R(I_1 \cap I_2) = R(I_1) \cap R(I_2)$ where $R(I)$ denotes the radical of an ideal I.

(2) Prove that a hyperlane $H = \{l = 0\} \subset \mathbb{P}^n$ is irreducible (here l is a linear homogeneous polynomial).

(3) Prove that a conic curve $C = \{x^2 + y^2 + z^2 = 0\} \subset \mathbb{P}^2$.

(4) Find the dimension of the space of homogeneous polynomials in n variables of degree d.

(5) Let V be a vector space such that $V = V_1 \oplus V_2$ for vector subspaces V_1 and V_2. Prove that a point $p \in \mathbb{P}(V) \setminus (\mathbb{P}(V_1) \cup \mathbb{P}(V_2))$ lies on a unique line $\mathbb{P}^1 \subset \mathbb{P}(V)$ that intersects both $\mathbb{P}(V_i)$.