MATH IN MOSCOW. ALGEBRAIC GEOMETRY. HOMEWORK 4

(1) Prove that a Veronese map $v_d\colon \mathbb{P}^n \to \mathbb{P}^{\binom{n+d}{n}-1}$ given by the formula
$$v_d(x_0 : \ldots : x_n) \mapsto (x_0^d : x_0^{d-1}x_1 : \ldots : x_n^d)$$
(take all monomials of degree d in x_i) is an isomorphism onto its image.

(2) Using Segre map $s\colon \mathbb{P}^n \times \mathbb{P}^m \to \mathbb{P}^{(n+1)(m+1)-1}$ given by the formula
$$s[(x_0 : \ldots : x_n), (y_0 : \ldots : y_m)] \mapsto (x_0y_0 : x_0y_1 : \ldots : x_ny_m)$$
define the structure of a projective variety on $\mathbb{P}^n \times \mathbb{P}^m$.

(3) Prove that the image of the Veronese map is not contained in a hyperplane. Prove that the image of the Segre map is not contained in a hyperplane.

(4) Prove that a subset $V \subset \mathbb{P}^n \times \mathbb{P}^m$ is closed if and only if V is given by a family of equations
$$F_i(x_0, \ldots, x_n, y_0, \ldots, y_m) = 0$$
where F_i is a polynomial homogeneous both in x_i and y_j. Formulate and prove a similar claim for closed subsets of $V \subset \mathbb{P}^n \times \mathbb{A}^m$.

(5) Let $\phi\colon X \to Y$ be regular map of quasi-projective varieties. Prove that the graph $\Gamma_\phi = \{(x, y) \in X \times Y \mid y = \phi(x)\}$ is a closed subset of $X \times Y$ isomorphic to X.

(6) Let V be a 3-dimensional vector space. Prove that the closed subset of $\mathbb{P}(S^2V^*) = \mathbb{P}^5$ that corresponds to plane conics of rank 1 is isomorphic to $v_2(\mathbb{P}^2)$.