

**MATH IN MOSCOW. ALGEBRAIC GEOMETRY.
HOMEWORK 7**

- (1) Construct cellular decomposition for $\text{Gr}(2, 5)$. Describe the cells geometrically and find their dimension.
- (2) Recall that $\text{Gr}(k, n) \simeq \text{Gr}(n - k, n)$. Show that under this duality, the affine cell $\sigma \subset \text{Gr}(k, n)$ that corresponds to a Young diagram A is mapped to an affine cell which corresponds to the transpose diagram A^T , that is, the diagram obtained by flipping A around a 45 degrees line running northwest-to-southeast.
- (3) * Show that the image of the Plücker embedding $p: \text{Gr}(k, n) \rightarrow \mathbb{P}^{\binom{n}{k}-1}$ is defined by algebraic equations. Write down these equations for $\text{Gr}(2, 5)$.
- (4) Compute the number of lines meeting 4 general lines in \mathbb{P}^3 .
- (5) *Find the number of lines on a smooth cubic surface $S \subset \mathbb{P}^3$.