

**MATH IN MOSCOW. ALGEBRAIC GEOMETRY.
HOMEWORK 1**

- (1) Give an example of a non-principal ideal in $\mathbb{C}[x, y]$ and in $\mathbb{Z}[x]$.
- (2) Let $V \subset \mathbb{A}^n$ and $W \subset \mathbb{A}^m$ be algebraic subsets. Prove that $V \times W \subset \mathbb{A}^{n+m}$ is an algebraic subset.
- (3) In the assumptions of the previous problem, prove that if V and W are irreducible, then $V \times W$ is irreducible as well.
- (4) In the assumptions of the previous problem, prove (or disprove) that $W = V(I(W))$.
- (5) Prove that the subset $\{x_1 - e^{x_2} = 0\} \subset \mathbb{A}_{\mathbb{C}}^2$ is not algebraic.