

**MATH IN MOSCOW. ALGEBRAIC GEOMETRY.
HOMEWORK 3**

- (1) Let I_1, I_2 be ideals in a ring R . Prove that $R(I_1 \cap I_2) = R(I_1) \cap R(I_2)$ where $R(I)$ denotes the radical of an ideal I .
- (2) Show that in a Noetherian ring, for an ideal I there exists a number N such that $R(I)^N \subset I \subset R(I)$.
- (3) Let X and Y be affine algebraic sets. Prove that any homomorphism $\phi: \mathbb{K}[Y] \rightarrow \mathbb{K}[X]$ has the form $\phi = f^*$ for a unique regular map $f: X \rightarrow Y$.
- (4) Find the dimension of the space of homogeneous polynomials in n variables of degree d .