MATH IN MOSCOW. ALGEBRAIC GEOMETRY. HOMEWORK 6

(1) Find singular points of the following curves. Draw pictures over $\mathbb{R}$.
   (a) $x^2 = x^4 + y^4$,
   (b) $xy = x^6 + y^6$,
   (c) $x^3 = y^2 + x^4 + y^4$,
   (d) $x^2y + xy^2 = x^4 + y^4$.

(2) Prove that any irreducible singular cubic curve $C \subset \mathbb{P}^3$ (up to a change of coordinates on $\mathbb{P}^2$) has the form $y^2z = x^3$ or $y^2z = x^3 + x^2z$.

(3) How many singular points may an irreducible quartic curve $C \subset \mathbb{P}^2$ have? (A quartic is a curve given by an equation of degree 4). Give examples.

(4) What is the maximum number of singular points of a (not necessarily irreducible) plane curve $C \subset \mathbb{P}^2$ of degree d.