

MATH IN MOSCOW. TOPOLOGY-2. HOMEWORK 2.
DUE TO 21 SEPTEMBER 2022

- (1) Construct a CW structure on $\mathbb{C}P^n$ and $\mathbb{C}P^\infty$.
- (2) Prove that, for the natural CW structure on $\mathbb{C}P^2$, the restriction of the characteristic map $\phi: D^4 \rightarrow \mathbb{C}P^2$ to the boundary ∂D^4 is the Hopf fibration: $\phi|_{\partial D^4}: \partial D^4 = S^3 \rightarrow \mathbb{C}P^1 = S^2$.
- (3) Prove that a CW-complex is connected if and only if it is path-connected.
- (4) Prove that any finite CW-complex X^n of dimension n can be embedded in \mathbb{R}^N where $N = (n+1)(n+2)/2$.
- (5) Find a topological space satisfying (W) axiom of a CW complex, but not (C) axiom. Find a space satisfying (C) but not (W).
- (6) Prove that S^∞ is contractible.