MATH IN MOSCOW. TOPOLOGY-3. HOMEWORK 3.
DUE TO 28 SEPTEMBER 2022

(1) Prove that $\pi_1(S^1) = \mathbb{Z}$.

(2) Prove that $\pi_n(X, x)$ is a group for any ≥ 1 (using explicit formulas).

(3) Let $f\colon (X, x) \to (Y, y)$ be a map of pointed spaces. Prove that the induced map

$$f_* = \pi_n(f)\colon \pi_n(X, x) \to \pi_n(Y, y)$$

is well-defined.

(4) Assume that X is path-connected. Then does $\pi_n(X, x)$ depend on the point x?

(5) Prove that the connecting homomorphism ∂ in the long exact sequence of homotopy groups is well-defined.

(6) Prove that the long exact sequence of homotopy groups is indeed exact.