

Topology I

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MiM, Fall 2020

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Office Hours: ??? Class Hours: ???

Course Description

An introduction to topology, including basic point-set topology, the classification of surfaces, and the theory of fundamental groups and covering spaces.

There will be two 80 minute online meetings per week, one of which will consist of the instructor explaining material and the other will consist of students discussing and presenting solutions to assigned exercises, with the assistance of the instructor.

Recommended reading

- M.A. Armstrong, Basic Topology
- O. Viro et al., Elementary Topology Problem Textbook

Topics

1. Basic notions: Open and closed sets in a topological space, continuity, homeomorphism, connectedness and compactness, theorem of Heine-Borel on compact subsets of \mathbf{R}^n
2. Topological constructions (product, disjoint union, wedge, cone, suspension, quotient spaces, cell spaces). Cell complexes.
3. Topological manifolds and their classification in dimension 2.
4. Vector fields on the plane. Generic singular points. The index of a plane vector field. Vector fields on surfaces. The Poincaré index theorem.
5. Curves in the plane, degree of a point with respect to a curve, Whitney index (winding number) of a curve, classification of immersions, the fundamental theorem of algebra?. Degree of a map of a circle into itself. Brouwer fixed point theorem. Jordan separation theorem.

6. Fundamental group (main properties, simplest computations), covering spaces. Algebraic classification of covering spaces (via subgroups of the fundamental group of the base).

Marking scheme

The final mark will be based on the results of presenting solutions to homework in the online seminars (20 %) and two longer written assignments in place of a midterm and final (40 % each).

Rough schedule

We shall begin with basic point set topology and a host of constructions and examples, proceed to fundamental groups and covering spaces, and finish with applications to the topology of curves in the plane and the classification of compact oriented surfaces.

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